VII Računske vežbe

1. U prostoriji dimenzija 20×10×10 m je instalirano 50 mašina iste zvučne snage. Vreme reverberacije prostorije sa 50 mašina je 2 s. Ako se zbog potreba tehnološkog procesa u istoj prostoriji montira još 100 istih mašina, odrediti za koliko će se povećati nivo buke u prostoriji ako je apsorpcija jedne mašine $0,5\ m^2$. (7. knjiga)

$a = 20\ m$
$b = 10\ m$
$c = 10\ m$
$\Big\}\ V = abc = \boxed{2000\ m^3}$

$n = 50$

$T_R = 2\ s, \quad T_R > 0,5 \Rightarrow$ Sabinov obrazac

P_a

$n' = 100$

$\Delta L = ?$

$A_m = 0,5\ m^2$ – apsorpciona površina svake mašine

$T_R = 0,162 \frac{V}{A} \Rightarrow A = 0,162 \frac{V}{T_R}$

$A = 0,162 \cdot \frac{2000}{2} = 0,162 \cdot 1000 = \boxed{162\ m^2} \Rightarrow \boxed{A = 162\ m^2}$

$I = \frac{4 \sum_i P_{ai}}{A} = \boxed{\frac{4 n P_a}{A}}$

$I' = \frac{4 \sum_i P_{ai}}{A'} = \boxed{\frac{4(n+n')P_a}{A + n' A_m}}$

$\Delta L = L' - L = 10 \log \frac{I'}{I_0} - 10 \log \frac{I}{I_0} = 10 \left(\log \frac{I'}{I_0} - \log \frac{I}{I_0} \right) =$

$= 10 \log \left(\frac{\frac{I'}{I_0}}{\frac{I}{I_0}} \right) = 10 \log \left(\frac{I'}{I} \right) = 10 \log \left(\frac{\frac{4(n+n')P_a}{A + n' A_m}}{\frac{4 n P_a}{A}} \right) =$

$= \boxed{10 \log \left[\frac{A(n+n')}{n(A + n' A_m)} \right]}$

$$\Delta L = 10\log\left(\frac{150\,A}{50(A+n'A_m)}\right) = \boxed{10\log\left(\frac{3A}{A+n'A_m}\right)} =$$

$$= 10\log\left(\frac{3\cdot 162}{162+100\cdot 0{,}5}\right) = 10\log\left(\frac{3\cdot 162}{162+50}\right) = 10\log\left(\frac{3\cdot 81}{106}\right) =$$

$$= \boxed{3{,}6\ dB} \Rightarrow \boxed{\Delta L = 3{,}6\ dB}$$

(2.) Dva izvora zvuka, zvučne snage po $10\,mW$, nalaze se u prostoriji zapremine $200\,m^3$. Prvi izvor zvuka emituje prost zvuk na frekvenciji 100 Hz, a drugi na frekvenciji 1000 Hz. Vreme reverberacije prostorije za frekvenciju od 100 Hz iznosi 2 s, a za frekvenciju od 1000 Hz iznosi 4 s. Odrediti:

(a) Rezultujući nivo zvuka u prostoriji

(b) Promenu ukupnog nivoa zvuka u prostoriji nakon 1 s od istovremenog prekida rada oba izvora. (12. knjiga)

$$P_a = 10\,mW = 10\cdot 10^{-3}\,W = 10^{-2}\,W$$

$$V = 200\,m^3$$

$$\left.\begin{array}{ll} f_1 = 100\,Hz & T_{R1} = 2\,s \\ f_2 = 1000\,Hz & T_{R2} = 4\,s \end{array}\right\} \begin{array}{l} T_{R1,2} > 0{,}8\,s \\ \Rightarrow \text{Sabinov obrazac} \end{array}$$

$$\boxed{T_R = 0{,}162\,\frac{V}{A}}$$

(a) $L_R = ?$

(b) $\Delta L_R = ?$

(a) $I_1 = \dfrac{25\,P_a T_{R1}}{V}$, $f_1 = 100\,Hz$

$$I_1 = \frac{25\cdot 10^{-2}\,W\cdot 2\,s}{200\,m^3}\cdot\frac{m}{s} = \boxed{2{,}5\cdot 10^{-3}\,\frac{W}{m^2}}$$

$$L_1 = 10\log\frac{I_1}{I_0} = 10\log\left(\frac{2{,}5\cdot10^{-3}\ \frac{W}{m^2}}{10^{-12}\ \frac{W}{m^2}}\right) = 10\log\left(2{,}5\cdot10^{9}\right) =$$

$$= 10\log(2{,}5) + 10\log 10^{9} = \underbrace{10\log(2{,}5)}_{3{,}9} + 90 = \boxed{93{,}9\ dB}$$

$f_2 = 1000\ Hz$, $I_2 = 2I_1$

$$I_2 = \frac{25\,\eta\,T_{R2}}{V} = \frac{25\cdot10^{-2}\cdot 4}{200}\ \frac{W}{m^2} = \boxed{5\cdot10^{-3}\ \frac{W}{m^2}}$$

$$L_2 = 10\log\frac{I_2}{I_0} = 10\log\left(\frac{5\cdot10^{-3}\ \frac{W}{m^2}}{10^{-12}\ \frac{W}{m^2}}\right) = 10\log\left(5\cdot10^{9}\right) =$$

$$= 10\log 5 + 10\log 10^{9} = \underbrace{10\log 5}_{6{,}9} + 90 = \boxed{96{,}9\ dB}$$

$$I_R = I_1 + I_2 = 2{,}5\cdot10^{-3}\ \frac{W}{m^2} + 5\cdot10^{-3}\ \frac{W}{m^2} = \boxed{7{,}5\cdot10^{-3}\ \frac{W}{m^2}}$$

$$L_R = 10\log\frac{I_R}{I_0} = 10\log\left(\frac{7{,}5\cdot10^{-3}\ \frac{W}{m^2}}{10^{-12}\ \frac{W}{m^2}}\right) = 10\log\left(7{,}5\cdot10^{9}\right) =$$

$$= 10\log(7{,}5) + 10\log 10^{9} = \underbrace{10\log(7{,}5)}_{8{,}7} + 90 = \boxed{98{,}7\ dB}$$

⑥ $f_1 = 100\ Hz$, $t = 1\ s$

$t : \Delta L_1 = T_{R1} : 60$

$$\Delta L_1 = \frac{60\,t}{T_{R1}} = \frac{60\cdot1}{2} = \boxed{30\ dB}$$

$f_2 = 1000\ Hz$

$t : \Delta L_2 = T_{R2} : 60$

$$\Delta L_2 = \frac{60\,t}{T_{R2}} = \frac{60\cdot1}{4} = \boxed{15\ dB}$$

$$L_1' = L_1 - \Delta L_1 = 93{,}9\ dB - 30\ dB = 63{,}9\ dB$$

$$L_2' = L_2 - \Delta L_2 = 96{,}9\ dB - 15\ dB = 81{,}9\ dB$$

$$L_R' = 10\log\left(\sum_{i=1}^{2} 10^{\frac{L_i'}{10}}\right) = 10\log\left[10^{\frac{L_1'}{10}} + 10^{\frac{L_2'}{10}}\right) =$$

$$= 10\log\left(10^{\frac{63{,}9}{10}} + 10^{\frac{81{,}9}{10}}\right) = 10\log\left(10^{6{,}39} + 10^{8{,}19}\right) = \boxed{81{,}9\ dB}$$

$$\Delta L_R = L_R - L_R' = 98{,}7\ dB - 81{,}9\ dB = \boxed{16{,}8\ dB}$$

(3) U proizvodnoj hali radi 50 mašina iste zvučne snage. Apsorpcija prazne hale iznosi $20\,m^2$, a prosečna apsorpcija svake mašine $0,2\,m^2$. Izračunati:

(a) koliko se još mašina može vneti v halv, a da se nivo ne poveča za više od 3 dB,

(6) koliko je teorijsko maksimalno povečanje nivoa bez obzira na broj vnetih mašina? (8. naloga)

$n = 50$

$A_0 = 20\,m^2$

$A = 0,2\,m^2$

$\Delta L = 3\,dB$

$n' = ?$

$\Delta L_{max} = ?$

(a) $\Delta L = L_2 - L_1 = 10\log\frac{I_2}{I_0} - 10\log\frac{I_1}{I_0} = 10\left(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0}\right) =$

$= 10\log\left(\frac{\frac{I_2}{I_0}}{\frac{I_1}{I_0}}\right) = 10\log\frac{I_2}{I_1} \Rightarrow \boxed{\Delta L = 10\log\frac{I_2}{I_1}}$

$\frac{\Delta L}{10} = \log\frac{I_2}{I_1} \Rightarrow \frac{I_2}{I_1} = 10^{\frac{\Delta L}{10}} = 10^{\frac{3}{10}} = 10^{0,3} = \boxed{2} \Rightarrow \boxed{I_2 = 2I_1}$

$I_1 = \frac{4P_{a1}}{A_1}$

$P_{a1} = nP_a$

$A_1 = A_0 + nA$

$\Big\} \Rightarrow \boxed{I_1 = \frac{4nP_a}{A_0 + nA}}$

$I_2 = \frac{4P_{a2}}{A_2}$

$P_{a2} = (n+n')P_a$

$A_2 = A_0 + (n+n')A$

$\Big\} \Rightarrow \boxed{I_2 = \frac{4(n+n')P_a}{A_0 + (n+n')A}}$

$I_2 = 2 I_1$

$$\Rightarrow \quad \frac{\cancel{4}(n+n')\cancel{P_a}}{A_0 + (n+n')A} = \frac{2 \cdot \cancel{4} n \cancel{P_a}}{A_0 + nA}$$

$$(n+n')(A_0 + nA) = 2n\,[A_0 + (n+n')A]$$

$$n(A_0 + nA) + n'(A_0 + nA) = 2n(A_0 + nA) + 2nn'A$$

$$n'(A_0 + nA) - 2nn'A = 2n(A_0 + nA) - n(A_0 + nA)$$

$$n'[A_0 + nA - 2nA] = n(2A_0 + 2nA - A_0 - nA)$$

$$n'(A_0 - nA) = n(A_0 + nA)$$

$$\boxed{n' = \frac{A_0 + nA}{A_0 - nA} \cdot n}$$

$$n' = \frac{20 + 50 \cdot 0{,}2}{20 - 50 \cdot 0{,}2} \cdot 50 = \frac{20 + 10}{20 - 10} \cdot 50 = \frac{30}{10} \cdot 50 = 3 \cdot 50 = 150$$

$\boxed{n' = 150 \text{ mašina}}$

⑥ $$\frac{I_2}{I_1} = \frac{\frac{\cancel{4}(n+n')\cancel{P_a}}{A_0 + (n+n')A}}{\frac{\cancel{4} n \cancel{P_a}}{A_0 + nA}} = \boxed{\frac{(n+n')(A_0 + nA)}{n(A_0 + (n+n')A)}}$$

$$\Delta L = 10 \log \frac{I_2}{I_1} = 10 \log \frac{(n+n')(A_0 + nA)}{n(A_0 + (n+n')A)}$$

$$\Delta L_{max} = \lim_{n' \to \infty} 10 \log \frac{I_2}{I_1} = 10 \log \lim_{n' \to \infty} \frac{I_2}{I_1} =$$

$$= 10 \log \lim_{n' \to \infty} \frac{(n+n')(A_0 + nA)}{n(A_0 + (n+n')A)}$$

$$\Rightarrow \Delta L_{max} = 10 \log \lim_{n' \to \infty} \frac{n'(A_0 + nA) + n(A_0 + nA)}{nA_0 + n^2 A + nn'A} =$$

$$= 10 \log \lim_{n' \to \infty} \frac{n' \left[A_0 + nA + \frac{n}{n'} (A_0 + nA) \right]}{n' \left[nA + \frac{n' A_0}{n'} + \frac{nA_0}{n'} \right]} =$$

$$= 10 \log \left(\frac{A_0 + nA}{nA} \right) = 10 \log \left(\frac{nA}{nA} + \frac{A_0}{nA} \right) = \boxed{10 \log \left(1 + \frac{A_0}{nA} \right)} =$$

$$= 10 \log \left(1 + \frac{20 \, m^2}{50 \cdot 0{,}2 \, m^2} \right) = 10 \log \left(1 + \frac{2}{1} \right) = 10 \log 3 =$$

$$= \boxed{4{,}77 \, dB}$$

$$\Rightarrow \boxed{\Delta L_{max} = 4{,}77 \, dB}$$

(4.) Dva izvora zvuka iste zvučne snage su instalirana u prostoriji sa lošim akustičnim karakteristikama gde emituju dva prosta zvuka na frekvencijama 100 Hz i 1000 Hz. Nakon isključenja, zapaženo je da je vreme opadanja nivoa zvuka do praga čujnosti isto za oba izvora. Prag čujnosti na frekvenciji 100 Hz je za 25 dB viši od praga čujnosti na 1000 Hz. Odrediti subjektivnu jačinu zvuka na 1000 Hz. Prosečni nivo zvuka na 100 Hz je za 3 dB viši od nivoa zvuka na 1000 Hz.

(11. knjiga)

$p_{a1} = p_{a2} = p_a$

$f_1 = 100 \, Hz \qquad \Delta L = L_1 - L_2 = 3 \, dB$

$f_2 = 1000 \, Hz \qquad \Delta L_0 = 25 \, dB$

Izofonska linija od 0 fona odgovara pragu čujnosti.

$$\Delta L = L_1 - L_2 = 10 \log \frac{I_1}{I_0} - 10 \log \frac{I_2}{I_0} = 10 \left(\log \frac{I_1}{I_0} - \log \frac{I_2}{I_0} \right) =$$

$$= 10 \log \left(\frac{\frac{I_1}{I_0}}{\frac{I_2}{I_0}} \right) = \boxed{10 \log \frac{I_1}{I_2}}$$

$$\log \frac{I_1}{I_2} = \frac{\Delta L}{10}$$

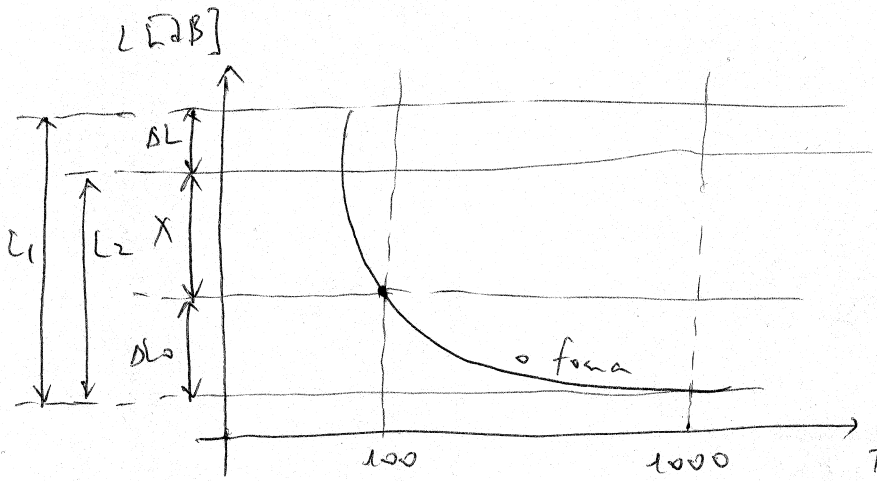
$$\frac{I_1}{I_2} = 10^{\frac{\Delta L}{10}} = 10^{\frac{3}{10}} = 10^{0,3} = 2 \Rightarrow \boxed{\frac{I_1}{I_2} = 2} \quad \boxed{I_1 = 2 I_2}$$

$$I = \frac{4 P_a}{A}$$

$$T_R = 0,162 \frac{V}{A} \Rightarrow A = 0,162 \frac{V}{T_R}$$

$$\Rightarrow I = \frac{4 P_a}{0,162 \frac{V}{T_R}} = \frac{25 P_a T_R}{V}$$

$$\left.\begin{aligned} I_1 &= \frac{25 P_a T_{R1}}{V} \\ I_2 &= \frac{25 P_a T_{R2}}{V} \end{aligned}\right\} \frac{I_1}{I_2} = \frac{\frac{25 P_a T_{R1}}{V}}{\frac{25 P_a T_{R2}}{V}} = \frac{T_{R1}}{T_{R2}} = 2$$



$f_1 = 100$ Hz nivo zvuka opadne za $(x + \Delta L)$ dB do praga čujnosti

$f_2 = 1000$ Hz nivo zvuka opadne za $(x + \Delta L_o)$ dB do praga čujnosti

Razlika praga čujnosti na 100 Hz i 1000 Hz iznosi $\Delta L_o = 25$ dB.

Nivo zvuka v prostoriji nakon isključenja izvora opada po linearnom zakonu, i da za vreme koje je jednako vremenu reverberacije nivo zvuka opadne za 60 dB.

$t : (x+\Delta L) = T_{R1} : 60$

$60t = T_{R1}(x+\Delta L)$

$T_{R1} = \frac{60t}{x+\Delta L}$

$t : (x+\Delta L_0) = T_{R2} : 60$

$60t = T_{R2}(x+\Delta L_0)$

$T_{R2} = \frac{60t}{x+\Delta L_0}$

$$\frac{T_{R1}}{T_{R2}} = \frac{\frac{60t}{x+\Delta L}}{\frac{60t}{x+\Delta L_0}} = \frac{x+\Delta L_0}{x+\Delta L}$$

$$\frac{T_{R1}}{T_{R2}}(x+\Delta L) = x+\Delta L_0$$

$$\frac{T_{R1}}{T_{R2}}x + \frac{T_{R1}}{T_{R2}}\Delta L = x+\Delta L_0$$

$$\frac{T_{R1}}{T_{R2}}x - x = \Delta L_0 - \frac{T_{R1}}{T_{R2}}\Delta L$$

$$x\left(\frac{T_{R1}}{T_{R2}} - 1\right) = \Delta L_0 - \frac{T_{R1}}{T_{R2}} \cdot \Delta L$$

$$\boxed{x = \frac{\Delta L_0 - \frac{T_{R1}}{T_{R2}}\Delta L}{\frac{T_{R1}}{T_{R2}} - 1}}$$

$\frac{T_{R1}}{T_{R2}} = 2$

$x = \frac{\Delta L_0 - 2\Delta L}{2-1} \Rightarrow \boxed{x = \Delta L_0 - 2\Delta L}$

$\left.\begin{array}{l}\Delta L_0 = 25\,dB \\ \Delta L = 3\,dB\end{array}\right\}$ $x = 25 - 2\cdot 3 = 25 - 6 = \boxed{19\,dB} \Rightarrow \boxed{x = 19\,dB}$

Nivo zvuka komponente na $f_2 = 2000\,Hz$ je:

$L_2 = \Delta L_0 + x = 25 + 19 = \boxed{44\,dB}$

Subjektivna jačina zvuka na $f_2 = 1000\,Hz$: $\boxed{L_2 = 44\ Fon}$

STIVENSENOV DIJAGRAM

Merenjem buke tercnim filtrom dobijeni su rezultati dati u tabeli.

f_0 [Hz]	40	50	63	80	100	125	160	200	250	315	400	500
L [dB]	62	68	72	74	72	70	60	64	74	76	68	64

Odrediti rezultujući nivo buke i subjektivnu jačinu složenog zvuka.

$L = ?,\ S = ?,\ \Lambda = ?$

f_0 [Hz]	40	50	63	80	100	125	160	200	250	315	400	500
L [dB]	62	68	72	74	72	70	60	64	74	76	68	64
$L = 10\log\sum_{i=1}^{3}\left(10^{L_i/10}\right)$ oktavni nivo	73,8			77,1			74,6			76,9		
$S(f, L)$ [son]	3,9			6,8			7,9			10,7		

$$\mathrm{L} = 10\log\sum_{i=1}^{12} 10^{\mathrm{L_i}/10} = 10\log(10^{\mathrm{L_1}/10} + 10^{\mathrm{L_2}/10} + 10^{\mathrm{L_3}/10} + 10^{\mathrm{L_4}/10}) = 81{,}8\ dB$$

$$S = S_m + F\left(\sum_{i=1}^{4} S_i - S_m\right)$$

$\mathrm{S_m} = \max(\mathrm{S_i}) = 10{,}7\ sona,$

$\mathrm{F} = 0{,}15 - tercni\ spektar$

$\mathrm{F} = 0{,}3 - oktavni\ spektar$

$\sum_{i=1}^{4} \mathrm{S_i} = 29{,}3\ sona$

$\mathrm{S} = 13{,}5\ sona$

$$\Lambda = 40 + \frac{10}{\log 2}\log \mathrm{S} = 77{,}5\ \ fona$$